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Fracture Studies in Rubber-Modified Acrylics. I. Experimental Method: Design of Sandwich-Tapered Double-Cantilever Beam Cleavage Specimens

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Synopsis

Double-cantilever beam cleavage specimens are very useful in characterizing the fracture behavior of isotropic, homogeneous, brittle, and semibrittle polymers. However, until this time they have not been used for ductile materials such as rubber-modified polymers or polycarbonates because of excessive yielding or breakage of the specimen arms before crack propagation can occur. In order to make it possible to measure the fracture surface work of these materials, a new sandwich cleavage specimen was developed by bonding rigid reinforcement plates to both sides of the specimen sheet. With this sandwich-tapered double-cantilever beam cleavage specimen, one can create conditions under which controlled crack propagation through tough ductile materials and measurement of the fracture surface work—two previously unobtainable results—are readily determined. In this paper, the design and construction of the sandwich specimens, test procedure, and the data analysis will be discussed in detail.

INTRODUCTION

Brittle glassy polymers often exhibit catastrophic fracture. Ductile and tough polymers are, therefore, selected or specially designed for applications requiring impact and fracture resistance. But, unlike brittle glassy polymers for which theories and methods to evaluate fracture toughness are well established, a quantitative method of evaluation of fracture toughness of ductile and tough polymers has not yet been developed. This presents a problem in deciding which one of several ductile and tough materials has the best resistance against fracture or spallation. In recent years, two-phase polymer systems (rubber-modified polymers) have been developed to improve upon the impact resistance of brittle glassy polymers. However, finding the rubber percentage or particle size to optimize toughness has been difficult since conventional testing methods such as notched Izod impact testing and puncture testing may give contradictory results and are rather arbitrary test techniques.[1]

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A double-cantilever beam cleavage specimen has been very useful in characterizing fracture toughness or fracture surface work for brittle and semi-brittle polymeric materials. Utilizing this type of specimen, the effects of various material parameters such as molecular weight, molecular weight distribution, crosslinking, crystallinity, and molecular structure on the fracture surface work have been successfully studied.[2] But until this time, the double-cantilever beam cleavage specimen has not yet been useful for ductile or tough materials because of excessive bending or breakage of the specimen arms. In this paper, a new technique for measurement of fracture surface work for ductile and tough polymers utilizing a sandwich cleavage specimen will be presented. The sandwich-tapered double-cantilever beam cleavage specimen has been used to study several commercial and experimental rubber-modified acrylics and found to be very successful in characterizing the effects of the rubber phase on fracture toughness as a function of crack velocity. Design and construction, test procedures, and data analysis for this specimen type will be discussed in detail in this paper, and experimental results and their analysis will be discussed in another paper (part II of this series).

DESIGN AND CONSTRUCTION OF A SANDWICH-TAPERED DOUBLE-CANTILEVER BEAM CLEAVAGE SPECIMEN

Since a sandwich-tapered double-cantilever beam cleavage specimen is a modification of the tapered cleavage specimen, the basic design of the latter type will be reviewed briefly. The use of this type of specimen has been reported originally by Mostovoy, Crossley, and Ripling.[3] The crack extension force or critical strain energy release rate, G_c, which is twice the value of fracture surface work γ and is frequently referred to in fracture toughness investigations, is defined as:

$$G_c = 2\gamma = \frac{f_c^2}{2w}\frac{\partial C}{\partial l} \tag{1}$$

where f_c = applied load at fracture, w = crack width, l = crack length, and C = total specimen compliance at crack length. If the specimen is designed so that the compliance changes linearly with crack length, i.e., $\partial c/\partial l$ = constant, then G_c or γ depends only upon the failure load f_c, providing that the crack width remains constant. Since the cleavage specimen is treated as a pair of identical cantilever beams, design of a tapered cleavage specimen is determined through the terms representing bending, shear, and end rotation deflections. The compliance of the specimen, C, can be expressed as $C = 2\delta/f$, where 2δ = total separation of cantilever beams at the point of loading and f = force applied to the specimen ends.

The equation which relates the fracture surface work to experimentally determined variables has been derived elsewhere and is shown below:[2,3]

$$G_c = 2\gamma = \frac{f_c^2}{2w}\frac{6M}{Eb} \tag{2}$$

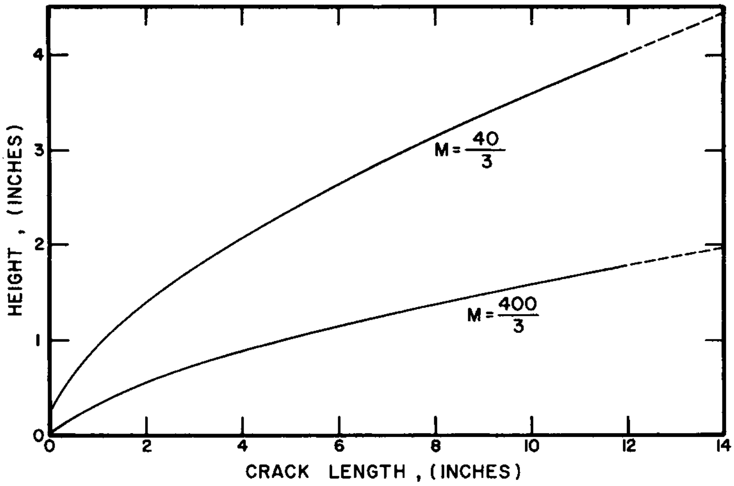

Fig. 1. Calculated tapered specimen contours for constants of $M = 400/3$ and $40/3$.

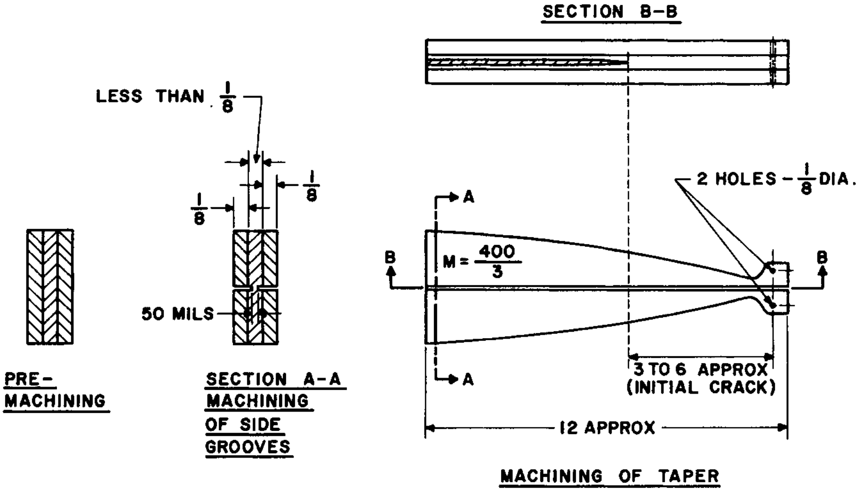

Fig. 2. Sandwich-tapered cleavage specimen construction steps.

where E = modulus of elasticity, b = specimen width, and M = a constant which governs the taper of the specimen. The relation between the length and height of the beam is

$$l = \left[\sqrt{Mh - (1 + \nu)} - 1.2\right]\frac{h}{2} \qquad (3)$$

where ν = Poisson's ratio. From eq. (3), for any given value of the constant M, a relation between h and l can be established. The specimens used in this study have been designed for an M of 400/3 assuming $\nu = 0.35$. The effect of altering the value of M is shown in Figure 1 and a tapered cleavage specimen is shown in Figure 2.

The specimen thus designed has several advantages over other types of specimens.

1. With a constant $\partial C/\partial l$ by design, the relation between fc and G_c or γ is independent of the crack length l. Thus, it is not necessary to measure load, deflection, and crack length continuously. This greatly simplifies testing and data analysis.
2. For constant deflection rates, one can obtain a constant crack length extension rate, providing that the crack width remains constant. In other words, crack velocity is a function of deflection rate. Thus, with this type of specimen it is possible to study fracture surface work as a function of crack velocity.
3. By determining $\partial C/\partial l$ experimentally, the change in Young's modulus for different testing rates is automatically taken into consideration. This is very useful in testing polymeric materials, since properties of polymeric materials are strain rate dependent.

The tapered double-cantilever beam cleavage specimen has been successfully applied to brittle and semibrittle glassy polymers, and the above advantages have been fully utilized to characterize fracture toughness of these materials. For ductile and tough materials, however, the tapered double-cantilever beam cleavage specimen is not valid. Ductile and tough polymeric materials normally exhibit low yielding stresses, high elongations, and large values of fracture surface work. Therefore, in order to propagate a crack through the tapered cleavage specimen, one has to apply a large load to the specimen, as can be seen from eq. (1). This high load results in the excess bending and yielding or breakage of the beams of the specimen without crack extension. The sandwich-tapered double-cantilever beam cleavage specimen was developed to eliminate this problem.

Construction of the sandwich-tapered double-cantilever beam cleavage specimen is as follows (Fig. 2).

1. Strong, elastic reinforcement plates are bonded to both sides of the specimen sheet by proper adhesives. Young's modulus of the reinforcement plates should be higher than that of the test material.
2. After bonding, the side grooves and initial crack are machined *through* the reinforcement plates *into* the test specimen by a specially designed side-groove cutting machine.[1]
3. The specimen is then affixed to a template with double-coated tape, and the contour of the tapered cleavage specimen is machined by a router.

The advantages of this specimen construction are further demonstrated by the following calculation. The compliance C_i of the test material made up in the form of a double-cantilever beam specimen, with crack length l, specimen thickness b_i, and Young's modulus E_i, is given by

$$C_i = \frac{2\delta_i}{f_i} = \frac{6}{E_i b_i}\left[\frac{4(l + l_0)^3}{3h^3} + \frac{(1 + \nu_i)l}{h}\right] \tag{4}$$

where $2\delta_i$ is the total deflection of the specimen end under the load f_i, ν_i is Poisson's ratio of test material, h is the height of the specimen at crack length l, and $l_0 = 0.6h$ (experimentally measured calibration correction factor). The compliance C_0 of the reinforcement material made up in the form of a double-cantilever beam specimen with crack length l, specimen thickness b_0, and Young's modulus E_0 is given by

$$C_0 = \frac{2\delta_0}{f_0} = \frac{6}{E_0 b_0}\left[\frac{4(l+l_0)^3}{3h^3} + \frac{1+\nu_0}{h}\,l\right] \tag{5}$$

where $2\delta_0$ is the total deflection of the specimen end under the load f_0, and ν_0 is Poisson's ratio of reinforcement material.

If the reinforcement plates in the shape of a double-cantilever beam specimen are bonded to both sides of a test material plate of the same shape, then the deflection of the three plates becomes the same. That is,

$$\delta_i = \delta_0 = \delta. \tag{6}$$

The total load f applied on the sandwich specimen end to produce deflection 2δ is

$$f = f_i + 2f_0. \tag{7}$$

Therefore, the compliance of the sandwich specimen is

$$C = \frac{2\delta}{f} = \frac{2\delta}{f_i + 2f_0} = \frac{1}{1/C_i + 2/C_0}. \tag{8}$$

From eqs. (4) and (5) we have

$$\frac{1}{C_i} + \frac{2}{C_0} = \frac{E_i b_i}{6}\left[\frac{1}{\dfrac{4(l+l_0)^3}{3h^3} + \dfrac{(1+\nu_i)}{h}\,l}\right] + \frac{2E_0 b_0}{6}\left[\frac{1}{\dfrac{4(l+l_0)^3}{3h^3} + \dfrac{(1+\nu_0)}{h}\,l}\right]. \tag{9}$$

If one neglects the difference between Poisson's rarios of the test material and the reinforcement material, the following can be obtained ($\nu_i = \nu_0 = \nu$):

$$\frac{1}{C_i} + \frac{2}{C_0} = \frac{E_i b_i + 2E_0 b_0}{6}\left[\frac{1}{\dfrac{4(l+l_0)^3}{3h^3} + \dfrac{(1+\nu)}{h}\,l}\right]. \tag{10}$$

Substituting eq. (10) into eq. (8), one obtains

$$C = \frac{2\delta}{f} = \frac{6}{E_i b_i + 2E_0 b_0}\left[\frac{4(l+l_0)^3}{3h^3} + \frac{(1+\nu)l}{h}\right] = \frac{6N}{E_i b_i + 2E_0 b_0}$$

$$\text{where } N = \left[\frac{4(l+l_0)^3}{3h^3} + \frac{(1+\nu)l}{h}\right]. \tag{11}$$

By differentiating eq. (11) with respect to l, the compliance change of the sandwich specimen, dC/dl, is obtained in the following form:

$$\frac{dC}{dl} = \frac{6}{E_ib_i + 2E_0b_0}\left[\frac{4(l + l_0)^2}{h^3} + \frac{(1 + \nu)}{h}\right] = \frac{6M}{E_ib_i + 2E_0b_0} \tag{12}$$

$$\text{where } M = \frac{4(l + l_0)^2}{h^3} + \frac{(1 + \nu)}{h} = \text{constant, by design.}$$

Equation (12) indicates that the compliance is independent of crack length. The crack extension force G is given by the same equation as eq. (1):

$$G = \frac{f^2}{2w}\frac{dC}{dl}. \tag{13}$$

From eqs. (11), (12), and (13) one now can calculate the crack extension force G for the case of the sandwich specimen with crack length l and given deflection $2\delta_g$. The resulting load f is from eq. (11):

$$f = \frac{2\delta_g(E_ib_i + 2E_0b_0)}{6N} \tag{14}$$

where N = a function of crack length and Poisson's ratio by design. The compliance change in terms of a crack length is from eq. (12):

$$\frac{dC}{dl} = \frac{6M}{E_ib_i + 2E_0b_0} \tag{15}$$

where M = constant by design. Therefore,

$$G = \frac{1}{2w}\left[\frac{2\delta_g(E_ib_i + 2E_0b_0)}{6N}\right]^2 \frac{6M}{E_ib_i + 2E_0b_0}.$$

Thus,

$$G = \frac{M(2\delta_g)^2(E_ib_i + 2E_0b_0)}{12N^2w}. \tag{16}$$

From eq. (16), one can see that for the given deflection δ_g the crack extension force can be increased by bonding reinforcement side plates with a higher Young's modulus, that is, by adding a large value of $2E_0b_0$ to E_ib_i. Furthermore, strong elastic reinforcement side plates increase the deflection capability of sandwich cantilever beams without yielding or breakage of the beams. Thus, if we select strong elastic reinforcement side plates with a higher Young's modulus than the test material, the combined effects increase significantly the value of crack extension force. One thereby creates conditions under which controlled crack propagation through ductile and tough materials and measurement of fracture surface work—two previously unobtainable factors—are readily obtainable. Other advantages of this sandwich-tapered cleavage specimen include the following: (1) It is particularly suitable for long-term environmental fracture tests because errors due to creep or stress relaxation of the cantilever beams are minimized by selecting metal reinforcement side plates. (2) In determining the crack velocity effect on the fracture surface work, this specimen is useful since, by changing the side reinforcement material, one can obtain a wide range of compliance changes and crack velocities.

In testing rubber-modified acrylics, cast Plexiglas sheets were selected for reinforcement side plates. Young's modulus of Plexiglas is approximately 1.3 to 2.3 times higher than that of the rubber-modified acrylics which were used as test materials. The stress–strain relation for the Plexiglas is nearly linear up to fracture, while the rubber-modified acrylics exhibit yielding at a much lower stress level.

TEST PROCEDURE

After machining, specimens are carefully oven annealed to eliminate the residual stress induced by machining. For the purposes of measuring crack velocities and obtaining the experimental value of the compliance change in terms of crack length, $\partial C/\partial l$, reference lines indicating the distances from the loading point of the specimen are drawn at half-inch intervals on the side of each annealed specimen. For low specimen separation rates performed in an Instron testing machine, these reference lines are utilized to mark the crack tip position by visual observation.

In the case of higher cross-head rate testing which was performed in a servo-controlled hydraulically actuated high-speed testing machine (maximum crosshead rate 18,000 in./min.), thin conductive paint lines are drawn over the reference lines which cross the crack path. When the crack breaks the conductive paint line, an electrical signal is produced. Load, specimen end displacement, and crack tip position signals are recorded by an oscilloscope as functions of time.

DATA ANALYSIS

The calculation of fracture surface work γ is accomplished by using eq. (1). In order to demonstrate the method of calculation of fracture surface work, the data for a rubber modified acrylic with 16% rubber will be analyzed. (This material was supplied to us by the U. S. Army Materials and Mechanics Research Center.) The load-versus-deflection curve for the sandwich specimen is shown in Figure 3. The measurement was made on an Instron testing machine using a separation rate of 0.05 inches per minute. Since the crack propagates in a continuous fashion, a continuous crack growth results with a nearly constant crack propagation load of 34 pounds. Any local variations of load are due to changes in crack width or area. Points are also shown on this graph representing various crack lengths determined visually from reference lines on the sample. It can be observed that the crack propagated at nearly constant velocity. In order to determine the experimental value of dC/dl, the compliance $C = 2\delta/f$ is calculated

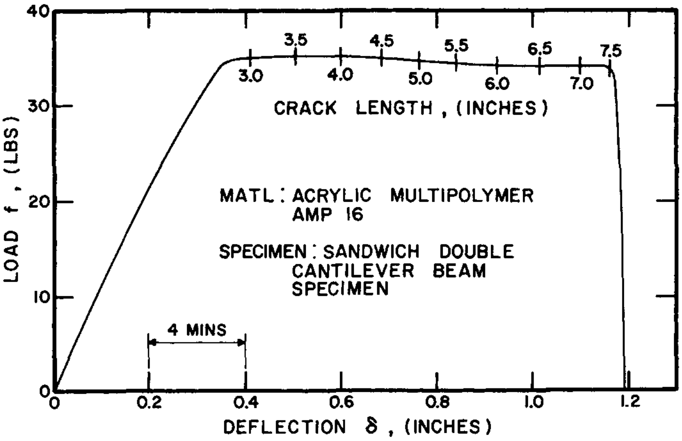

Fig. 3. Load-deflection curve from a sandwich double-cantilever beam cleavage specimen.

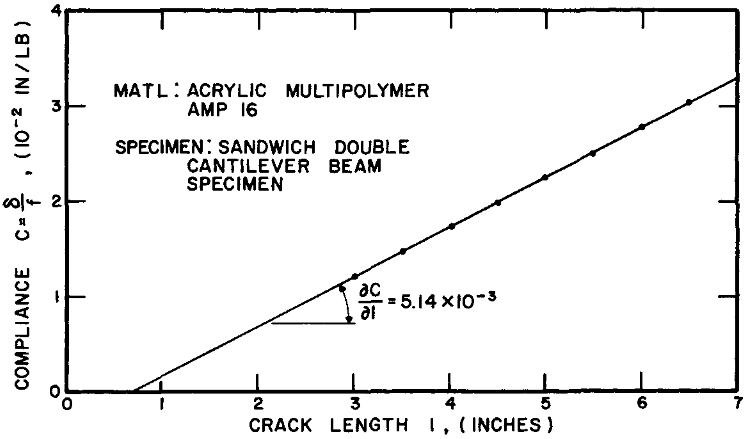

Fig. 4. Compliance-vs.-crack length plot for determination of experimental compliance change in terms of crack length.

at marked crack lengths in Figure 3, and the results are plotted in Figure 4. Figure 4 indicates that the compliance of this sandwich specimen is a linear function of crack length. The slope of the line in Figure 3 is the experimental value of $dC/dl = 5.14 \times 10^{-3}$. The measured crack width averaged 0.0524 in. By substituting these values in eq. (1), one can obtain the value of fracture surface work $\gamma = 28.4$ lb/in. $= 49.6 \times 10^5$ erg/cm^2.

The crack velocity is obtained from the load-deflection record shown in Figure 3 which has crack tip positions recorded as a function of time. For example, a crack propagated between $l = 6$ in. and $l = 7$ in. in 3.50 min; therefore, the crack velocity is 1.0 in./3.50 min = 0.286 in./min.

The critical strain energy release rate (G_c) or fracture surface work (γ) can be also calculated directly from the force–deflection curve of Figure 3.

This is accomplished by measuring the strain energy U released as the crack changes length, or

$$U = \frac{f(\delta_2 - \delta_1)}{2} \tag{17}$$

and equating this to the work done,

$$W = 2(l_2 - l_1)w\gamma \tag{18}$$

as the crack increases in length l_1 to l_2.

Thus,

$$\gamma = \frac{f(\delta_2 - \delta_1)}{4w(l_2 - l_1)} \tag{19}$$

and choosing a crack length increase from 6.0 in. to 7.0 in., the energy release is

$$U = \frac{34}{2}\,(1.100 - 0.925).$$

This work done is

$$W = 2(7.0 - 6.0) \times 0.0524 \times \gamma$$

so that

$$\gamma = \frac{34}{4 \times 0.0524}\,\frac{(1.100 - 0.925)}{(7.0 - 6.0)} = 28.4 \text{ lb/in.} = 49.6 \times 10^5 \text{ erg/in.}^2$$

This value agrees well with the one obtained by using eq. (1).

CONCLUSIONS

The specimen design and construction technique described in this paper can be used to determine the fracture surface work or toughness of any ductile, tough polymer. Actual experimental results for a series of rubber-modified acrylics obtained with this technique and their analysis will be discussed in part II of this series.

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